ANSWER KEY

1.	B
2.	A
3.	B
4.	C
5.	C
6.	C
7.	C
8.	D
9.	B
10.	A
11.	B
12.	A
13.	C
14.	D
15.	C
16.	D
17.	B
18.	D
19.	B
20.	C

SOLUTION

1. $\log_{49} 28 = \frac{\log 28}{\log 49} = \frac{\log 7 + \log 4}{2\log 7}$

$= \frac{\log 7}{2\log 7} + \frac{\log 4}{2\log 7} = \frac{1}{2} + \frac{1}{2}\log_7 4$

$= \frac{1}{2} + \frac{1}{2}.2\log_7 2 = \frac{1}{2} + \log_7 2$

$= \frac{1}{2} + m = \frac{1+2m}{2}$

2. $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$

$= \frac{1}{2}\log_e(ab) = \log_e\sqrt{ab}$

$\Rightarrow \frac{a+b}{2} = \sqrt{ab} \Rightarrow a+b = 2\sqrt{ab}$

$\Rightarrow \left(\sqrt{a}-\sqrt{b}\right)^2 = 0 \Rightarrow \sqrt{a}-\sqrt{b} = 0 \Rightarrow a = b$.

3. Since 10, 3, *e*, 2 are in decreasing order. Obviously, $\log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$ are in increasing order.

4. $\log_k x.\log_5 k = \log_x 5 \Rightarrow \log_5 x = \log_x 5$

$\Rightarrow \log_x 5 = \frac{1}{\log_x 5}$

$\Rightarrow (\log_x 5)^2 = 1$

$\Rightarrow \log_x 5 = \pm 1$

$\Rightarrow x^{\pm 1} = 5 \quad \Rightarrow x = 5, \frac{1}{5}$.

5. $\log_5 a.\log_a x = 2$

$\Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25$.

6. $a^2 + 4b^2 = 12ab$

$\Rightarrow a^2 + 4b^2 + 4ab = 16ab$

$\Rightarrow (a+2b)^2 = 16ab$

$\Rightarrow 2\log(a+2b) = \log 16 + \log a + \log b$

$\therefore \log(a+2b) = \frac{1}{2}[\log a + \log b + 4\log 2]$

7. $A = \log_2 \log_2 \log_4 256 + 2\log_{2^{1/2}} 2$

$= \log_2 \log_2 \log_4 4^4 + 2 \times \frac{1}{(1/2)}\log_2 2$

$= \log_2 \log_2 4 + 4 = \log_2 \log_2 2^2 + 4$

$= \log_2 2 + 4 = 1 + 4 = 5$.

8. $\log_{1000} x^2 = \log_{10^3} x^2$

$= 2\log_{10^3} x = \frac{2}{3}\log_{10} x = \frac{2}{3}y$.

9. $x = \log_a bc$

$\Rightarrow 1 + x = \log_a a + \log_a bc = \log_a abc$

$\therefore (1+x)^{-1} = \log_{abc} a$

$\therefore (1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc = 1$.

10. $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k\,(\text{say})$

$\Rightarrow \log x = k(b-c),\ \log y = k(c-a),$

$\log z = k(a-b)$

$\Rightarrow x = e^{k(b-c)}, y = e^{k(c-a)}, z = e^{k(a-b)}$

$\therefore\ xyz = e^{k(b-c)+k(c-a)+k(a-b)} = e^0 = 1$

$x^a y^b z^c = e^{k(b-c)a+k(c-a)b+k(a-b)c}$

$= e^0 = 1 = xyz$

$x^{b+c}y^{c+a}z^{a+b} = e^{k(b^2-c^2)+k(c^2-a^2)+k(a^2-b^2)}$

$= e^0 = 1.$

11. Let $\log_{16} x = y \Rightarrow y^2 - y + \log_{16} k = 0$

This quadratic equation will have exactly one solution if its discriminant vanishes.

$\therefore\ (-1)^2 - 4.1.\log_{16} k = 0 \Rightarrow 1 = \log_{16} k^4$

$\Rightarrow\ k^4 = 16\ \Rightarrow\ k^2 = 4\ \Rightarrow\ k = \pm 2.$

But $\log_{16} k$ is not defined $k < 0$, $\therefore k = 2$.

$\therefore$ Number of real values of $k = 1$.

12. $x = \log_5 1000 = 3\log_5 10 = 3 + 3\log_5 2$

$= 3 + \log_5 8$

$y = \log_7 2058 = \log_7(7^3.6)$

$= 3 + \log_7 6$

As $\log_5 8 > \log_5 5$ *i.e.*, $\log_5 8 > 1$.

$\therefore\ x > 4$

And $\log_7 6 < \log_7 7$ *i.e.*, $\log_7 6 < 1$

$\therefore\ y < 4;\ \therefore\ x > y.$

13. $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = \log_7 \log_7 7^{7/8}$

$= \log_7(7/8)$

$= \log_7 7 - \log_7 8 = 1 - \log_7 2^3$

$= 1 - 3\log_7 2.$

14. $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$

$= 3^{4\log_3 5} + 3^{3.\frac{1}{2}\log_3 36} + 3^{4\log_9 7}$

$= 3^{\log_3 5^4} + 3^{\log_3 36^{3/2}} + 3^{\log_3 7^{4/2}}$

$= 5^4 + 36^{3/2} + 7^2 = 890$

15. Given expression

$= \log\left(\frac{16^7}{15^7}.\frac{25^5}{24^5}.\frac{81^3}{80^3}\right) = \log 2.$

16. Consider $\log_3 [\log_3[\log_3 x]] = \log_3 3$

$\Rightarrow\ \log_3 [\log_3 x] = 3$

$\Rightarrow\ \log_3 x = 3^3$

$\Rightarrow\ \log_3 x = 27 \Rightarrow x = 3^{27}$

17. Let $\log(a + \sqrt{a^2+1}) + \log\left(\frac{1}{a + \sqrt{a^2+1}}\right)$

$= \log(a + \sqrt{a^2+1}) + \log 1 - \log(a + \sqrt{a^2+1})$

$= \log(a + \sqrt{a^2+1}) - \log(a + \sqrt{a^2+1})$

$= 0$

18. Consider, $\log \frac{9}{8} - \log\frac{27}{32} + \log\frac{3}{4}$

$= \log\frac{9}{8} + \log\frac{32}{27} + \log\frac{3}{4}$

$= \log\left(\frac{9}{8}\times\frac{32}{27}\right) + \log\frac{3}{4}$

$= \log\left(\frac{4}{3}\right) + \log\frac{3}{4} = \log\left(\frac{4}{3}\times\frac{3}{4}\right) = \log 1 = 0$

19. $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$,

$$\frac{\log x}{\log 3} \times \frac{\log 2x}{\log x} \times \frac{\log y}{\log 2x} = \frac{\log x^2}{\log x}$$

$$\frac{\log y}{\log 3} = \frac{2\log x}{\log x}$$

$\log y = 2 \log 3$

$\log y = \log 9$

$\boxed{y = 9}$

20. $\log_{10} 2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in A.P.

Hence, common difference will be same.

$\therefore \log_{10}(2^x - 1) - \log_{10} 2$

$= \log_{10}(2^x + 3) - \log_{10}(2^x - 1)$

$$\therefore \log_{10}\left(\frac{2^x - 1}{2}\right) = \log_{10}\left(\frac{2^x + 3}{2^x - 1}\right)$$

$$\Rightarrow \frac{2^x - 1}{2} = \frac{2^x + 3}{2^x - 1}$$

$(2^x - 1)^2 = 2(2^x + 3)$

$2^{2x} - 2^{x+1} + 1 = 2^{x+1} + 6$

$2^{2x} - 2^{x+2} = 5$

Let $2^x = y$, then

$y^2 - 4y - 5 = 0$

$y^2 - 5y + y - 5 = 0$

$y(y - 5) + 1(y - 5) = 0$

$y = -1, y = 5$

Therefore, $2^x = 5$

$x = \log_2 5$.